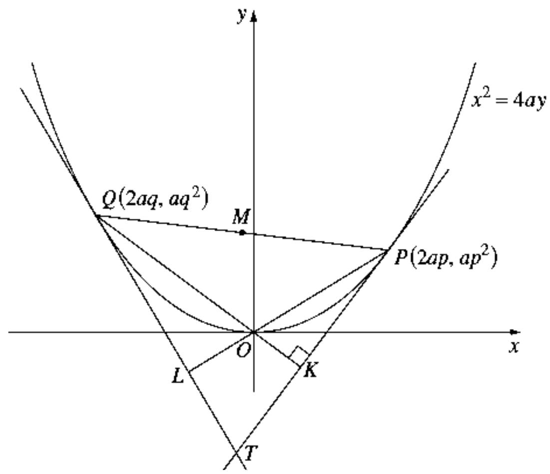
08	**4c**	The points $P\ (2ap, ap^2)$, $Q\ (2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The tangents to the parabola at P and Q intersect at T. The chord QO produced meets PT at K, and $\angle PKQ$ is a right angle. (i) Find the gradient of QO, and hence show that $pq = -2$. (ii) The chord PO produced meets QT at L. Show that $\angle PLQ$ is a right angle. (iii) Let M be the midpoint of the chord PQ. By considering the quadrilateral $PQLK$, or otherwise, show that $MK = ML$. 	**2** **1** **2**

(i) Using $Q(2aq, aq^2)$ and $O(0, 0)$:

$$M_{QO} = \frac{aq^2 - 0}{2aq - 0} = \frac{q}{2}$$

Now, finding gradient of PT:

$$x^2 = 4ay$$
$$y = \frac{x^2}{4a}$$
$$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$$

At $P(2ap, ap^2)$: $\frac{dy}{dx} = \frac{2ap}{2a} = p$

$\therefore\ m_{PT} = p$

But using $PT \perp QO$: $p \times \frac{q}{2} = -1$

$pq = -2$

(ii) As $m_{QO} = \frac{q}{2}$ (from (i)), then

$$M_{PO} = m_{PL} = \frac{p}{2}$$

Also, as $m_{PT} = p$ (from (i)), then $M_{QL} = q$

Now using $m_{PL} \times m_{QL} = \frac{p}{2} \times q$

$$= \frac{pq}{2}$$
$$= \frac{-2}{2} \text{ (from (i))}$$
$$= -1$$

$\therefore\ m_{PL} \times m_{QL} = -1$, then $PL \perp QL$

$\therefore\ \angle PLQ = 90^\circ$

(iii) $\angle PKQ = 90^\circ$ $\angle PLQ = 90^\circ$

$\therefore$ $QLKP$ is cyclic quadrilateral

($\angle$s in same sector equal)

$\therefore$ QP is diameter

($\angle$s in semi circle are right angles)

$\therefore$ M is centre of circle

$\therefore$ $MK = ML$ (equal radii)

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Well set out solutions with clear logic were more successful. Mid-range responses found the gradient of QO but then many multiplied it by $\frac{p}{2}$ stating $\frac{p}{2} \times \frac{q}{2} = -1$ and therefore $pq = -2$.

(ii) It was important to relate the results in (c)(i) to this part. The most successful method simply stated that $\frac{p}{2} \times q = \frac{pq}{2} = \frac{-2}{2} = -1$. Some candidates who could not establish the result in part (i) nevertheless used the result to successfully complete part (ii).

(iii) In the better responses, candidates who recognised that $PQLK$ was a cyclic quadrilateral were quite efficient and effective at explaining why $ML = MK$. Those who tried coordinate geometry formulae found that it was nearly impossible to prove the result and so they spent valuable time completing large amounts of algebra to little benefit.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/