Solutions

2014

Year 12 Mathematics

Trial HSC Examination

Teacher Setting Paper: Mrs T Finch
Head of Department: Mrs M Hill

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
  (graphics calculators are not allowed)
- A table of standard integrals is provided as part of this examination
- If using writing paper (instead of examination booklets), write on one side of the paper only and write your student number at the top of every page

Total Marks – 100

Section I Pages 2 – 5

10 Marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 13

90 Marks

- Attempt Questions 11 – 16
- Each Question is worth 15 marks
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Allow about 2 hours and 45 minutes for this section
- Hand in each question separately

This examination paper does not necessarily reflect the content or format of the Higher School Certificate Examination in this subject

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

QUESTION 1

What is $\sqrt[3]{6\frac{2}{7}}$ written correct to 3 significant figures?

(A) 1.197

(B) 1.20

(C) 1.846

(D) 1.85

QUESTION 2

What is the perpendicular distance from $(2,-3)$ to the line $y=4x-2$

(A) $\frac{3}{\sqrt{17}}$

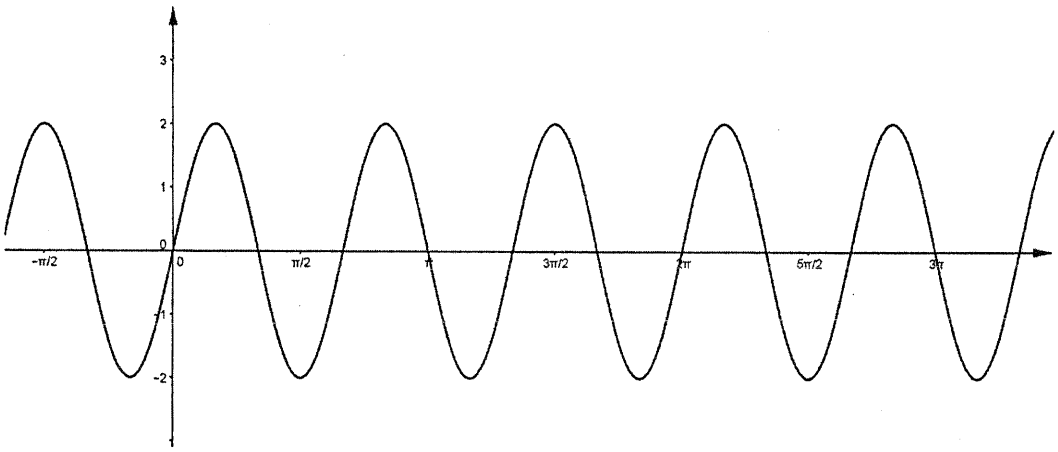
(B) $\frac{3}{\sqrt{15}}$

(C) $\frac{9}{\sqrt{17}}$

(D) $\frac{9}{\sqrt{15}}$

QUESTION 3

The graph shows the function:



(A) $y = 3\sin 2x$

(B) $y = 2\sin 3x$

(C) $y = 3\sin\left(\frac{x}{2}\right)$

(D) $y = 2\sin\left(\frac{x}{3}\right)$

QUESTION 4

If $\frac{2}{\sqrt{10}+2} = x\sqrt{10} + y$, what are the values of x and y ?

(A) $x = \frac{1}{7},\ y = \frac{2}{7}$

(B) $x = \frac{1}{7},\ y = -\frac{2}{7}$

(C) $x = \frac{1}{4},\ y = -\frac{1}{2}$

(D) $x = \frac{1}{3},\ y = -\frac{2}{3}$

QUESTION 5

$\int \frac{1}{3x}\,dx = ?$

(A) $\ln\frac{x}{3}+c$

(B) $\frac{1}{3}\ln x+c$

(C) $3\ln x+c$

(D) $\ln 3x+c$

QUESTION 6

An ordinary pack of cards is shuffled. Andy chooses two cards from the pack at the same time. Find the probability that the cards are both clubs.

(A) $\frac{13}{52}\times\frac{13}{52}$

(B) $\frac{13}{52}+\frac{13}{52}$

(C) $\frac{13}{52}\times\frac{12}{51}$

(D) $\frac{13}{52}+\frac{12}{51}$

QUESTION 7

Evaluate $\sum_{k=5}^{32}(3k-1)$

(A) 1526

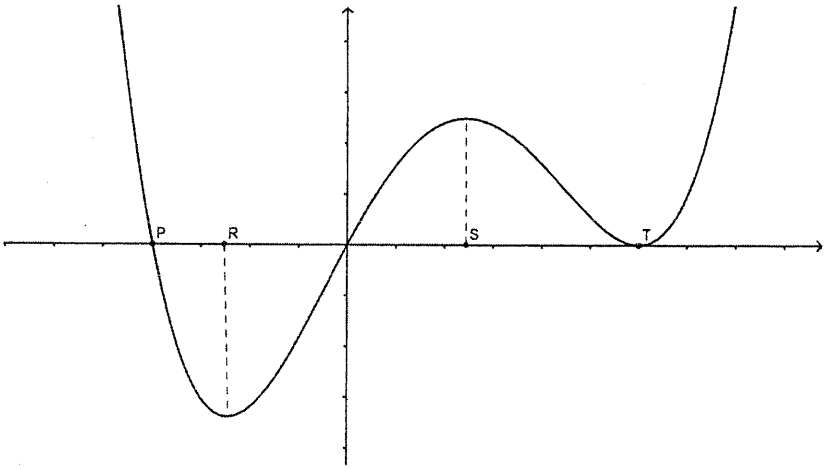
(B) 1471.5

(C) 1269

(D) 95

QUESTION 8

The diagram shows the graph of $y = f'(x)$.



Which letter indicates the position of the x-value of the maximum turning point?

(A) P

(B) R

(C) S

(D) T

QUESTION 9

A parabola has the equation $x^2 + 2x + 25 = 8y$. The focus of the parabola is:

(A) $(-1,1)$

(B) $(-1,3)$

(C) $(-1,5)$

(D) $(-1,11)$

QUESTION 10

The fourth term of a Geometric series is 192 and the seventh term is – 24. What is the infinite sum of the series?

(A) 1024

(B) – 1024

(C) 128

(D) – 128

Section II is on the next page

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet (hand each question in separately)
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

QUESTION 11 (15 Marks) Hand this question in separately. **Marks**

(a) Simplify $\dfrac{1}{x^2-1}-\dfrac{1}{x+1}$ **2**

(b) Differentiate the following functions, leaving your answers in simplest form:

(i) $3x^2-\sin 2x$ **2**

(ii) $4x\sqrt{x^2-1}$ **3**

(c) An arc of $2cm$ subtends an angle of θ at the centre of a circle of radius $14cm$.
Find the value of θ, correct to the nearest degree. **1**

(d) Solve $|4-2x|=12x$, showing working. **3**

(e) The quadratic equation $3x^2-2x+6=0$ has roots α and β. Find

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{2}{\alpha}+\dfrac{2}{\beta}$ **1**

(iv) $\alpha^2+\beta^2$ **1**

QUESTION 12 (15 Marks) Hand this question in separately. **Marks**

(a) Solve $\tan^2 x = 3$ for $-\pi \leq x \leq \pi$ **2**

(b) Differentiate $y = \dfrac{\ln x}{x^2}$ with respect to x **2**

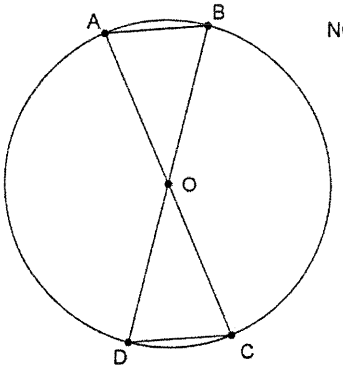
(c) Find the equation of the tangent to the curve $y = 3e^{2x}$ at the point where $x = 1$ **3**

(d) Find:

(i) $\displaystyle\int \frac{2x-1}{x^2-x}\,dx$ **2**

(ii) $\displaystyle\int_0^{\pi} \sin\frac{x}{2}\,dx$ **2**

(e) O is the centre of the circle radius $10cm$.



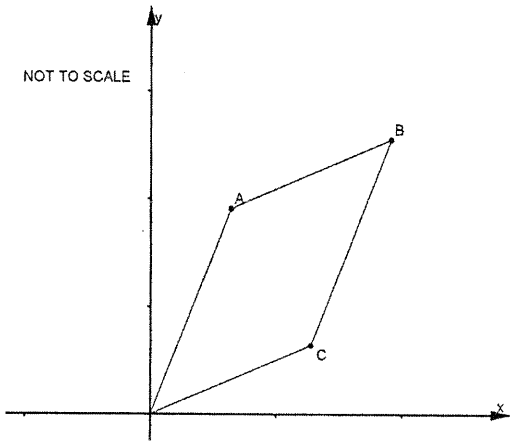
NOT TO SCALE

(i) Prove the triangles OAB and ODC are congruent. **2**

(ii) If $\angle AOB = \dfrac{\pi}{5}$, find in exact form, the area of sector OBC. **2**

QUESTION 13 (15 Marks) Hand this question in separately. **Marks**

(a) The diagram shows the parallelogram $OABC$. Points O, A and C are $(0,0)$, $(2,6)$ and $(4,2)$ respectively.



(i) Find the length of the interval OC. **1**

(ii) Find the equation of the line passing through O and C. **1**

(iii) Find the midpoint of AC. **1**

(iv) Hence, or otherwise, find the co-ordinates of B. **1**

(v) Find the perpendicular distance from A to OC. **1**

(vi) Find the area of the parallelogram $OABC$. **1**

(b) Find the values of k for which the quadratic equation $x^2 - kx + k + 8 = 0$

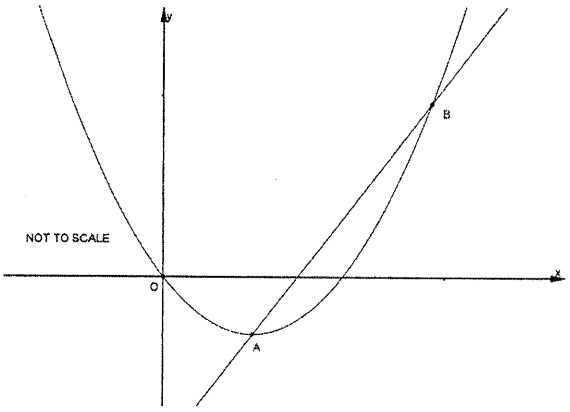
(i) Has equal roots **1**

(ii) Has two real roots **2**

Question 13 is continued on the next page.

QUESTION 13 (continued)

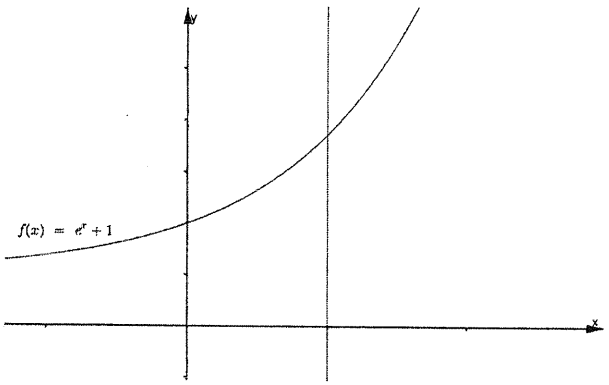
(c) The curve $y = x^2 - 2x$ and the straight line $2x - y - 3 = 0$ intersect at the points A and B as shown.



(i) Find the x – co-ordinates of A and B. **1**

(ii) Find the area contained between the straight line and the curve. **2**

(d) In the diagram the curves $y = e^x + 1$ and $x = 1$ are shown.

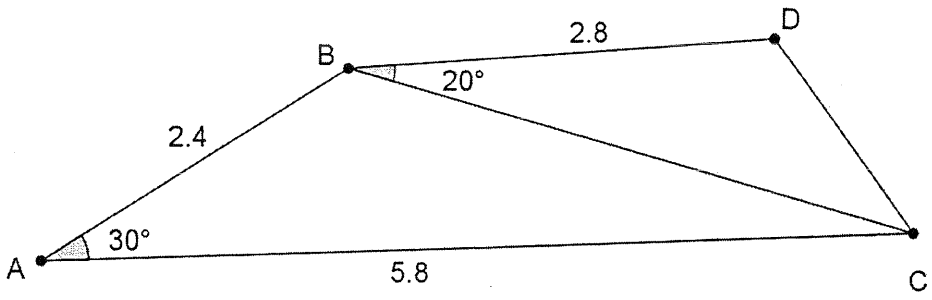


The region bounded by the curve $y = e^x + 1$, the x-axis, the y-axis and the line $x = 1$ is rotated about the x-axis.

Find the exact volume of the solid formed. **3**

QUESTION 14 (15 Marks) Hand this question in separately. **Marks**

(a) In the diagram $AB = 2.4m$, $AC = 5.8m$, $BD = 2.8m$, $\angle BAC = 30°$ and $\angle DBC = 20°$



NOT TO SCALE

(i) Show that the length of $BC = 3.9m$, to 1 decimal place. **2**

(ii) Hence, or otherwise, find the area of triangle BCD, to the nearest square metre. **2**

(b) A school basketball team has a probability of 0.75 of losing or drawing a match

(i) Find the probability of the school team **winning** at least one of 4 consecutive matches, correct to 2 decimal places. **1**

(ii) What is the least number of matches the team must play to be 95% certain of winning at least one match? **2**

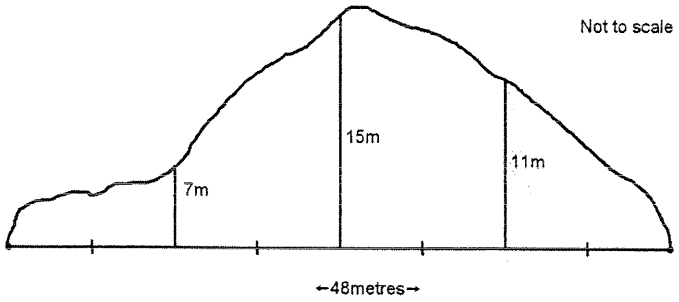
Question 14 is continued on the next page.

QUESTION 14 (continued)

(c) Given that $\sin\theta = \frac{2}{3}$ and $\cot\theta < 0$, find the exact value for $\cos\theta - \tan\theta$, making sure that you fully simplify your answer. **1**

(d) Solve the equation $3^{2x} - 7\left(3^{x}\right) - 18 = 0$ **2**

(e) A pond in Cook Park is bordered by a straight path on one side which is 48 metres in length, and a curve on the other edges as shown. The measurements are in metres.



Using Simpson's rule, find the approximate area of the pond surface, in exact form. **2**

(f) A student decides to save money over one year. In her first week she puts aside 10c. In the second week 40c, in the third week 70c, and so on with constant increases over time.

(i) What amount will she put aside in her 52nd week? **1**

(ii) How much has she saved altogether over the one year? **2**

QUESTION 15 (15 Marks) Hand this question in separately. **Marks**

(a) (i) Prove the identity $\frac{1}{\sin\theta+1}-\frac{1}{\sin\theta-1}=2\sec^2\theta$ **2**

(ii) Hence find $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\left(\frac{1}{\sin\theta+1}-\frac{1}{\sin\theta-1}\right)d\theta$ in exact form. **2**

(b) A vessel initially contains 100 litres of liquid. It is being emptied and the rate of change of the Volume is $\frac{dV}{dt}=-\left(2+\frac{20}{t+1}\right)$ where V is the volume in litres after a time of t minutes.

(i) At what rate is the vessel emptying initially? **1**

(ii) Find how many litres of liquid remain in the vessel after 5 minutes. Answer correct to 3 significant figures. **3**

(c) Julian borrowed \$20 000 from a finance company to purchase a car. Interest on the loan is calculated quarterly at the rate of 10% pa and is charged immediately prior to Julian making his quarterly payment of \$M.

(i) Write an expression for A_1 for the amount owing after 1 payment has been made. **1**

(ii) Show that $A_n=20000\times1.025^n-40M\left(1.025^n-1\right)$ **2**

(iii) If the loan were to be paid out after 7 years, what would the value of M be? **2**

(iv) If Julian were to pay \$1282.94 per quarter in repayments, how long would it take to pay out his loan? **2**

QUESTION 16 (15 Marks) Hand this question in separately. **Marks**

(a) A particle is moving with acceleration $\ddot{x} = 6t \ cm\,s^{-2}$. Initially it is at the origin and has velocity 12 $cm\,s^{-1}$ in the negative direction.

(i) Show that the equation for the velocity of the particle is $\dot{x} = 3t^2 - 12$ **1**

(ii) Show that the particle is stationary when $t = 2$ **1**

(iii) Hence find its maximum distance from the origin on the negative side. **1**

(iv) When does it return to the origin? **1**

(v) How far has the particle travelled after 5 seconds? **1**

(vi) Describe the particle's position and velocity as time goes on. **1**

(b) Consider the curve $y = x^3 - 6x^2 + 9x - 4$

(i) Find any stationary points and determine their nature **3**

(ii) Find the co-ordinates of the point of inflexion. **1**

(iii) Hence, sketch the curve, showing all turning points, the y-intercept and the point of inflexion. **1**

Question 16 is continued on the next page.

QUESTION 16 (continued)

(c) The mass M kg of a radioactive substance present after t years is given by the equation $M = M_0 e^{-kt}$, where k is a positive constant.

After 50 years the substance has been reduced from $20kg$ to $10kg$ in mass.

(i) Show that $\frac{dM}{dt} = -kM$ **1**

(ii) State the value of M_0 **1**

(iii) Find the exact value of k. **1**

(iv) Find the time taken for the substance to lose 80% of its original mass. Answer to the nearest year. **1**

End of examination

Student Number _____

2014
Year 12 Mathematics
Trial HSC Examination

MULTIPLE CHOICE
ANSWER SHEET

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

This page may be removed from the question pages.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ
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8. Ⓐ Ⓑ Ⓒ Ⓓ
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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq 1;\quad x\neq 0, \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right)\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x>0$

Solutions
2014 Yr12 Trial Maths.

I

1	D	5	B
2	C	6	C
3	B	7	A
4	D	8	A
9	C		
10	B		

1. $\sqrt[3]{6\frac{2}{7}} = 1.8455..$
≈ 1.85

2. $4x - y - 2 = 0 \quad (2, -3)$

$$d = \frac{|4\times 2 - 1\times -3 - 2|}{\sqrt{4^2 + (-1)^2}}$$

$$= \frac{9}{\sqrt{17}}$$

3. $y = 2\sin 3x$

4. $\frac{2}{\sqrt{10}+2} \times \frac{\sqrt{10}-2}{\sqrt{10}-2}$

$$= \frac{2\sqrt{10}-4}{10-4}$$

$$= \frac{2(\sqrt{10}-2)}{6_3}$$

$x = \frac{1}{3} \quad y = -\frac{2}{3}$

5. $\int \frac{1}{3x}\,dx = \frac{1}{3}\int \frac{1}{x}\,dx$

$= \frac{1}{3}\ln x + c$

6. $P(CC) = \frac{13}{52} \times \frac{12}{51}$

7. $\sum_{R=5}^{32} (3k-1) \qquad n = 32-5+1 = 28 \text{ terms}$

$= 14 + 17 + 20 + \cdots + 95$

$S_n = \frac{n}{2}(a+l)$

$= \frac{28}{2}(14+95)$

$= 1526$

8. Max $\Rightarrow y' > 0, \ y' = 0, \ y' < 0$

$\therefore P$

9. $x^2 + 2x + 25 = 8y$

$x^2 + 2x + 1 = 8y - 25 + 1$

$(x+1)^2 = 8y - 24$

$(x+1)^2 = 8(y-3)$

$4a = 8 \quad V(-1, 3)$

$a = 2 \quad \text{focus } (-1, 5)$

10. $T_4 = 192 \qquad T_7 = -24$

$ar^3 = 192$
$ar^6 = -24$

$\frac{ar^6}{ar^3} = \frac{-24}{192}$

$r^3 = -\frac{1}{8}$

$r = -\frac{1}{2}$

$a \times (-\frac{1}{2})^3 = 192$

$a = -1536$

$S_\infty = \frac{-1536}{1 - -\frac{1}{2}} = -1024 \quad D$

Section II

11 a) $\frac{1}{x^2-1} - \frac{1}{x+1}$

$= \frac{1}{(x-1)(x+1)} - \frac{1}{(x+1)} \times \frac{(x-1)}{(x-1)}$ ✓

$= \frac{1-(x-1)}{(x-1)(x+1)}$ ✓

$= \frac{2-x}{(x-1)(x+1)}$

b) i) $\frac{d}{dx}(3x^2 - \sin 2x)$

$= 6x - 2\cos 2x$ ✓

ii) $\frac{d}{dx}\left(4x\sqrt{x^2-1}\right)$

$u = 4x$, $u' = 4$, $v = (x^2-1)^{1/2}$, $v' = \frac{1}{2}(x^2-1)^{-1/2} \times 2x = x(x^2-1)^{-1/2}$

$= 4x \times \frac{x}{(x^2-1)^{1/2}} + (x^2-1)^{1/2} \times 4$ ✓

$= \frac{4x^2(x^2-1)^{1/2}}{x^2-1} + \frac{4(x^2-1)(x^2-1)^{1/2}}{x^2-1}$ ✓

$= \frac{(x^2-1)^{1/2}\left[4x^2 + 4x^2 - 4\right]}{x^2-1}$

$= \frac{\sqrt{x^2-1}\,(8x^2-4)}{x^2-1}$ ✓

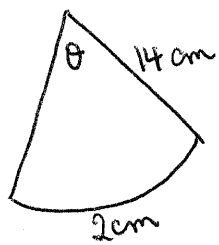
OR

$\frac{4x^2}{(x^2-1)^{1/2}} + \frac{4(x^2-1)^{1/2} \times (x^2-1)^{1/2}}{(x^2-1)^{1/2}}$

$= \frac{4x^2 + 4(x^2-1)}{(x^2-1)^{1/2}}$

$= \frac{8x^2-4}{(x^2-1)^{1/2}}$

c)



$l = \frac{\theta}{360} \times 2\pi r$

$2 = \frac{\theta}{360} \times 2 \times \pi \times 14$

$\theta = \frac{2 \times 360}{2 \times \pi \times 14}$

$= 8.185$ ✓

$\approx 8^\circ 11'$

(d) $|4-2x| = 12x$

$4-2x = 12x$ or $4-2x = -12x$

$4 = 14x$ $\quad 4 = -10x$

$x = \frac{4}{14}$ $\quad x = -\frac{4}{10}$

$= \frac{2}{7}$ ✓ $\quad = -\frac{2}{5}$

Test LHS = $\left|4-2\times\frac{2}{7}\right|$

$= 3\frac{3}{7}$

RHS $= 12 \times \frac{2}{7}$ ✓

$= 3\frac{3}{7}$ True

$\therefore x = \frac{2}{7}$

LHS $= \left|4-2\times-\frac{2}{5}\right|$

$= 4\frac{4}{5}$

RHS $= 12 \times -\frac{2}{5}$ ✓

$= -4\frac{4}{5}$

LHS ≠ RHS

e) $3x^2 - 2x + 6 = 0$

i) $\alpha + \beta = \frac{-2}{3} = \frac{2}{3}$ ✓

ii) $\alpha\beta = \frac{6}{3} = 2$ ✓

iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\beta + 2\alpha}{\alpha\beta}$

$= \frac{2(2/3)}{2} = \frac{2}{3}$ ✓

iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= \left(\frac{2}{3}\right)^2 - 2 \times 2 = -3\frac{5}{9}$ $\left(-\frac{32}{9}\right.$

12

a) $\tan^2 x = 3 \quad -\pi \le x \le \pi$ ✓✓

$\tan x = \pm\sqrt{3}$ ✓

$x = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$ ✓

b) $y = \frac{\ln x}{x^2}$

$u = \ln x \quad u' = \frac{1}{x}$

$v = x^2 \quad v' = 2x$

$y' = \frac{x^2 \times \frac{1}{x} - \ln x \times 2x}{(x^2)^2}$ ✓

$= \frac{x - 2x\ln x}{x^4}$

$= \frac{1 - 2\ln x}{x^3}$ ✓

c) $y = 3e^{2x} \quad y = 3e^2$ ✓

$y' = 6e^{2x}$

$= 6e^2$ ✓

$y - 3e^2 = 6e^2(x-1)$

$y - 3e^2 = 6e^2 x - 6e^2$ ✓

$y = 6e^2 x - 3e^2$

d) i) $\int \frac{2x-1}{x^2-x}\,dx$

$= \ln(x^2 - x) + c$ ✓✓

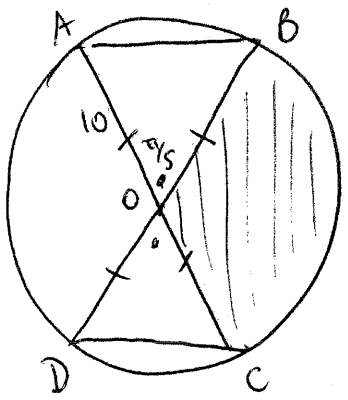
ii) $\int_0^{\pi} \sin\left(\frac{x}{2}\right) dx$

$= \left[-2\cos\frac{x}{2}\right]_0^{\pi}$ ✓

$= -2\cos\frac{\pi}{2} + 2\cos 0$

$= 0 + 2 = 2$ ✓

e)



$OA = OD$ } equal radii
$OB = OC$ } ✓

$\angle AOB = \angle DOC$ Vertically opposite angles

$\therefore \Delta OAB \equiv \Delta ODC$ SAS ✓

i) $\angle BOC = \pi - \frac{\pi}{5}$

$= \frac{4\pi}{5}$ ✓

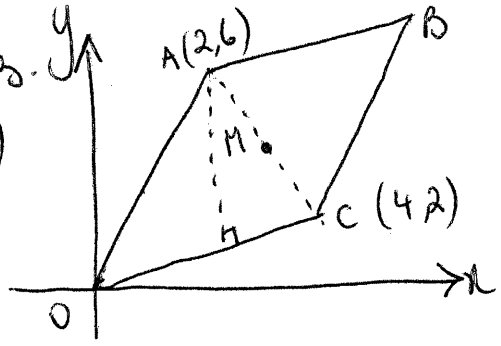
Area $= \frac{1}{2} r^2 \theta^c$

$= \frac{1}{2} \times 100 \times \frac{4\pi}{5}$ ✓

$= 40\pi \text{ cm}^2$

Q13.

a)



i) $d_{OC} = \sqrt{4^2 + 2^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$ ✓

ii) $m_{OC} = \frac{2}{4} = \frac{1}{2}$

$y = \frac{1}{2}x$ ✓

iii) $M_{AC} = \left(\frac{2+4}{2}, \frac{6+2}{2}\right)$

$= (3, 4)$ ✓

iv) $B(6, 8)$ ✓

v) $y = \frac{1}{2}x$

$2y = x$

$x - 2y = 0$

$d = \frac{|1 \times 2 - 2 \times 6 + 0|}{\sqrt{1^2 + (-2)^2}}$

$= \frac{10}{\sqrt{5}}$ ✓

vi) Area $= \overline{OC} \times d$

$= 2\sqrt{5} \times \frac{10}{\sqrt{5}}$

$= 20\ u^2$ ✓

b) $x^2 - kx + k + 8 = 0$

i) equal roots $\Delta = 0$

$(-k)^2 - 4 \times 1 \times (k+8) = 0$

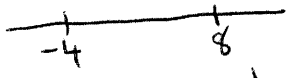
$k^2 - 4k - 32 = 0$

$(k-8)(k+4) = 0$

$k = 8$ or -4 ✓

ii) 2 real roots $\Delta > 0$

$(k-8)(k+4) > 0$



✓ $k < -4$ or $k > 8$ ✓

c) i) $y = x^2 - 2x$ ①

$2x - y - 3 = 0$ ②

$2x - (x^2 - 2x) - 3 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$ ✓

$x = 1$

$x = 3$

ii) Area $= \int_1^3 2x - 3 - (x^2 - 2x)\,dx$

$= \int_1^3 4x - 3 - x^2\,dx$ ✓

$= \left[\frac{4x^2}{2} - 3x - \frac{x^3}{3}\right]_1^3$ ✓

$= \left[(18 - 9 - 9) - \left(2 - 3 - \frac{1}{3}\right)\right]$

$= \frac{4}{3}\ u^2$

d) $V = \pi \int_0^1 (e^x + 1)^2\,dx$

$= \pi \int_0^1 e^{2x} + 2e^x + 1\,dx$ ✓

$= \pi \left[\frac{e^{2x}}{2} + 2e^x + x\right]_0^1$ ✓

$= \pi \left[e^2 + 2e + 1 - \left(\frac{1}{2} + 2\right)\right] = \pi\left(\frac{e^2}{2} + 2e - \frac{3}{2}\right)$ ✓

14. a) $BC^2 = 2.4^2 + 5.8^2 - 2 \times 2.4 \times 5.8 \times \cos 30$ ✓

$BC^2 = 15.28985\ldots$

$BC = \sqrt{15.28985\ldots}$

$= 3.910224\ldots$ ✓

≈ 3.9

ii) b) Area $= \frac{1}{2} \times 2.8 \times 3.9 \sin 20°$ ✓

$= 1.8723255\ldots$

$\approx 2\ m^2$ ✓

b) $P(\text{lose}) = 0.75$

$P(\text{win}) = 0.25$

i) $P(\text{at least one of 4})$

$= 1 - P(LLLL)$

$= 1 - 0.75^4$ ✓

$= 0.68$

ii) $0.95 = 1 - 0.75^n$ ✓

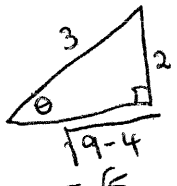
$0.75^n = 0.05$

$n \ln 0.75 = \ln 0.05$

$n = 10.41$ ✓

Need to play 11 matches

c) $\sin\theta = \frac{2}{3}$



(*✓) | ✓

*

$\cos\theta - \tan\theta$

$= -\frac{\sqrt{5}}{3} - -\frac{2}{\sqrt{5}}$

$= -\frac{\sqrt{5}}{3} + \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ ✓

$= -\frac{\sqrt{5}}{3} + \frac{2\sqrt{5}}{5}$

$= \frac{-5\sqrt{5} + 6\sqrt{5}}{15} = \frac{\sqrt{5}}{15}$

d) $3^{2x} - 7(3^x) - 18 = 0$ $\quad u = 3^x$

$u^2 - 7u - 18 = 0$

$(u-9)(u+2) = 0$ ✓

$u = 9$ or $u = -2$

$3^x = 9$ $\quad 3^x \neq -2$ as $3^x > 0$

$x = 2$ ✓

e) $A = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3) + 2y_2\right]$

$= \frac{12}{3}\left[0 + 0 + 4(7+11) + 2 \times 15\right]$

$= 4(72 + 30)$ ✓

$= 40\ m^2$

f) $10 + 40 + 70 + \cdots$

$a = 10 \quad d = 30$

i) $n = 52 \quad T_{52} = 10 + 51 \times 30$

$= 1540$ cents

or $= \$15.40$ ✓

ii) $S_{52} = \frac{52}{2}(10 + 1540)$ ✓

$= 40300$

$= \$403$ ✓

15a) i) $\frac{1}{\sin\theta+1} - \frac{1}{\sin\theta-1}$

$= \frac{(\sin\theta-1)-(\sin\theta+1)}{(\sin\theta+1)(\sin\theta-1)}$ ✓

$= \frac{-2}{\sin^2\theta - 1}$ $\quad \sin^2\theta+\cos^2\theta=1$, $\cos^2\theta = 1-\sin^2\theta$

$= \frac{-2}{-\cos^2\theta}$ ✓

$= 2\sec^2\theta$

ii) $\int_{-\pi/3}^{\pi/3} 2\sec^2\theta \, d\theta$ ✓

$= \left[2\tan\theta\right]_{-\pi/3}^{\pi/3}$

$= 2\tan\frac{\pi}{3} - 2\tan\left(-\frac{\pi}{3}\right)$ ✓

$= 2\sqrt{3} + 2\sqrt{3}$

$= 4\sqrt{3}$

b) $\frac{dV}{dt} = -\left(2 + \frac{20}{t+1}\right)$

i) Rate $= -(2+20)$

$= -22$ L/min ✓

ii) $\frac{dV}{dt} = -2 - \frac{20}{t+1}$ ✓

$V = -2t - 20\ln(t+1) + c$

$100 = 0 - 20\ln 1 + c$

$V = -2t - 20\ln(t+1) + 100$ ✓

$V = -2\times5 - 20\ln 6 + 100$

$V = 54.2$ L ✓

(c) $r = 0.1$ pa

$= 0.025$ per quarter

i) $A_1 = 20000 + 0.025\times20000 - M$

$A_1 = 20000(1.025) - M$ ✓

ii) $A_2 = A_1\times1.025 - M$

$A_2 = 20000(1.025)^2 - M\times1.025 - M$ ✓

$A_3 = 20000(1.025)^3 - M(1.025)^2$

$- M\times1.025 - M$

$A_n = 20000(1.025)^n - M(1.025^{n-1} + \dots + 1)$

$A_n = 20000(1.025)^n - M\left[\frac{1(1.025^n - 1)}{1.025 - 1}\right.$

$A_n = 20000(1.025)^n - M\left[\frac{1.025^n - 1}{\frac{1}{40}}\right.$ ✓

$A_n = 20000(1.025)^n - M\left[40(1.025^n - 1)\right.$ ✓

$A_n = 20000(1.025)^n - 40M(1.025^n - 1)$

(iii) $0 = 20000(1.025)^{28} - 40M(1.025^{28} - 1)$ ✓

$40M(1.025^{28} - 1) = \frac{20000(1.025)^{28}}{40(1.025^{28} - 1)}$

$M = \$1001.76$ ✓

iv) $0 = 20000(1.025)^n - 40\times1282.94\times(1.025^n - 1)$

$0 = 20000(1.025)^n - 51317.6\times1.025^n + 51317.6$

$31317.6\times1.025^n = 51317.6$ ✓

$1025^n = \frac{51317.6}{31317.6}$

$n\ln 1.025 = \ln\left(\frac{51317.6}{31317.6}\right)$

$n = 20.0000519$

≈ 20 payments ✓

16 a) $\ddot{x} = 6t$ $\quad t=0\ x=0\ \dot{x}=-12$

i) $\dot{x} = \frac{6t^2}{2} + c$

$-12 = 0 + c$ ✓

$\dot{x} = 3t^2 - 12$

ii) $\dot{x} = 3\times 2^2 - 12$

$= 0$ ∴ st. ✓

iii) $x = \frac{3t^3}{3} - 12t + c$

$0 = 0 - 12\times 0 + c$

$x = t^3 - 12t$

$x = 2^3 - 12\times 2$

$= 8 - 24$ ✓

$= -16$ cm

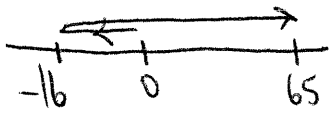
iv) $0 = t^3 - 12t$

$0 = t(t^2 - 12)$

$t = \sqrt{12}$ s ✓

v) $x = 5^3 - 12\times 5$

$= 65$



dist $= 2\times 16 + 65$

$= 97$ cm ✓!

vi) $t \to \infty$, $x \to \infty$

$\dot{x} \to \infty$

Starts at origin, moves to left initially & stops at 16cm then Moves to the right increasing its velocity and acceration towards the right. Accelerating. ✓

(b) $y = x^3 - 6x^2 + 9x - 4$

$y' = 3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$x = 1, 3$ ✓

$y'' = 6x - 12$

$x = 1$

$y'' = 6 - 12$

$= -6 < 0$ Concave down

∴ (1,0) is a Max ✓

$x = 1$

$y = 1^3 - 6 + 9 - 4$

$= 0$

$x = 3$

$y = 27 - 6\times 9 + 9\times 3 - 4$

$= -4$

$x = 3$ ✓

$y'' = 6\times 3 - 12$

$= 6 > 0$ Concave Up

∴ (3,-4) is Min

ii) $y'' = 0$

$6x - 12 = 0$

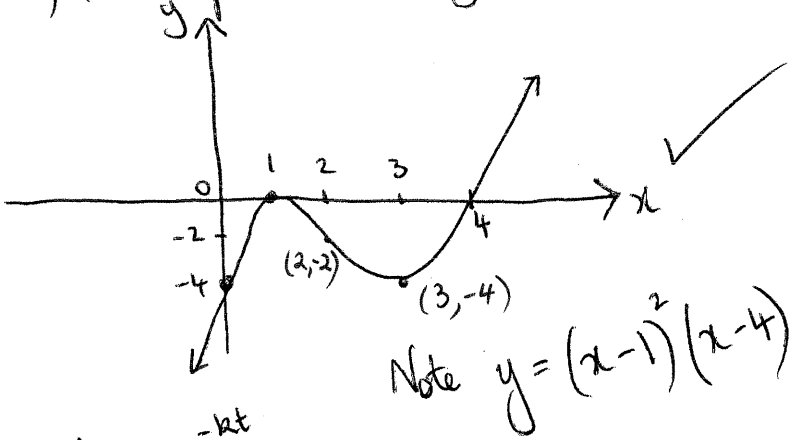
$x = 2$

$y = 2^3 - 6\times 2^2 + 9\times 2 - 4$

$= -2$

Inflexion at (2,-2) ✓

iii) Intercept $x=0$ $y=-4$



✓

Note $y = (x-1)^2(x-4)$

c) $M = M_0 e^{-kt}$

$\frac{dM}{dt} = -k\times M_0 e^{-kt}$ ✓

$= -kM$

ii) $M_0 = 20$kg

$-k\times 50$

iii) $10 = 20\, e$ ✓

$\ln\frac{1}{2} = -50k$

$k = -\frac{1}{50}\ln(\frac{1}{2}) = \frac{\ln 2}{50}$ ✓ ≈ 0.0139 /yr

iv) 20% of 20kg $= 4$kg

$4 = 20\, e^{-\frac{\ln 2}{50}t}$

$\ln 0.2 = -\frac{\ln 2}{50}t$ ✓

$t = \frac{-50\ln 0.2}{\ln 2}$

$= 116.096$ years

∴ in 117th year.