EXT – Motion (in terms of x)

J&C text

EXERCISE 26.2

1. Given $\dot{x} = \sqrt{6x}$, find the acceleration $\ddot{x}$.
2. A particle moves so that when it is x metres from the origin its velocity is given by $v = 2x - 1$. Find the acceleration of the particle when $x = 4$.
3. A particle moves with velocity v metres per second where $v^2 = 2x + 4$ and x metres is the displacement from the origin.
   (a) Find the acceleration of the particle.
   (b) Find the displacement of the particle when the velocity is zero.
   (c) Find the velocity of the particle when the displacement is 6 m. How do you explain the two answers?
4. A particle is moving in a straight line with a constant acceleration of 2 m/s^2 If it starts from the origin with a velocity of -6 m/s find:
   (a) an expression for the velocity in terms of the displacement.
   (b) its position when the velocity is zero.
   (c) the velocity of the particle when it returns to the origin.
5. A body starts 15 m from the origin, having a velocity of 12 m/s towards the origin. If the body is subjected to a constant acceleration of 3 m/s^2 in the opposite direction to its initial motion find:
   (a) an expression for the velocity in terms of the displacement.
   (b) the velocity when the displacement is 2·5 m.
6. A particle moves such that when it is x metres from the origin its acceleration is x^2 m/s^2. If initially the particle is at rest 2 m from the origin, find its velocity when it is 4 m from the origin.
7. A particle moves such that its acceleration when it is x metres from the origin is given by $\ddot{x} = \frac{1}{x}$. If the velocity is 2 m/s when $x = 1$, find the velocity when $x = e^2$
8. The motion of a particle is such that when it is x metres from the origin its acceleration is given by $a = -e^{-x}$ Given that $v = 2$ when $x = 0$, find v when $x = 2$.
9. A particle moving in a straight line has an acceleration given by $\ddot{x} = \dfrac{x}{x^2 - 8}$ when its displacement is x metres from the origin. Find its velocity when $x = 4$, given that $\dot{x} = 0$ when $x = 3$.
10. A particle moves such that when its displacement is x metres from the origin its velocity is given by $\dfrac{dx}{dt} = v = \sqrt{2x + 4}$.
    (a) By using the result that $\dfrac{dt}{dx} = \dfrac{1}{\frac{dx}{dt}}$, show that $t = \displaystyle\int (2x + 4)^{-\frac{1}{2}} dx$.
    (b) Given that $t = 0$ when $x = 0$, show that $t = \sqrt{2x + 4} - 2$ and hence that $x = \dfrac{t^2 + 4t}{2}$.
    (c) Find an expression for the velocity in terms of t and find the velocity when $t = 5$.
11. If the velocity of a particle is given by $v = \sqrt{4x + 25}$ where x metres is the displacement from the origin and if $t = 0$ when $x = 0$ find:
    (a) an expression for x in terms of t
    (b) an expression for v in terms of t.

solutions

1. $\ddot{x} = 3$
2. 14 m/s^2
3. (a) 1 m/s^2
   (b) -2 m
   (c) ± 4 m/s
4. (a) $v = 2\sqrt{x + 9}$
   (b) $x = -9$ m
   (c) $v = +6$ m/s
5. $v = \sqrt{6x + 54}$, $\pm 8{\cdot}3$ m/s
6. $v = 6{\cdot}1$ m/s
7. $v = 2{\cdot}8$ m/s
8. $v = 1{\cdot}5$ m/s
9. $v = \pm 1{\cdot}4$ m/s
10. (c) $v = t + 2$, 7 m/s
11. (a) $x = t^2 + 5t$
    (b) $v = 2t + 5$